UNSTABLE MHD CONTINUA IN MAGNETIZED ACCRETION DISKS.

Abstract. We review recent insights on magnetohydrodynamic waves and instabilities in accretion disk models. We apply MHD spectroscopy, i.e. the ability to compute all waves and instabilities for a given equilibrium configuration, and use the knowledge that the continuous parts (Alfvén and slow) in the spectrum form the basic organizing structure for categorizing the various wavetypes. We compute exact axisymmetric MHD disk tori, and demonstrate the existence of convective continuum instabilities in strongly magnetized accretion tori. This represents a new route to MHD turbulence in disks.

MHD spectroscopy. When adopting an ideal MHD description for accretion disk plasmas, the computation of all linear eigenmodes for a given disk equilibrium represents a forward seismological problem. In realistic accretion disk equilibrium, the plasma is gravitationally stratified, rotating, and will be thermally structured and pervaded by magnetic fields. In the last few years, we have made significant progress in the daunting task of categorizing the various MHD wave modes for these stationary, stratified, and magnetized configurations. In particular, it became obvious that magnetized accretion disks have many more linear routes to potentially magnetoturbulent dynamics than the prevailing interpretation based on the magneto-rotational instability (MRI, see Velikhov, 1959; Balbus & Hawley 1991). Moreover, even the latter instability can manifest itself quite differently in weak versus strongly magnetized disks, in the sense that its driving mechanism may change character from magneto-rotational to predominantly convective. We here briefly review some of these findings, emphasizing the various newly discovered instabilities along the way.

Magnetoseismology for radially stratified disks. In the analytically most tractable case, the radial stratification of the disk satisfies

$$(p + B_0^2 + B_z^2 / 2) \dot{\rho} = \rho \left( \frac{v^2}{r} + \frac{GM}{r^2} \right) - \frac{B_0^2}{r},$$

where the prime denotes differentiation with respect to the radial coordinate $r$ from a $(r, \theta, z)$ cylindrical system. Equation (1) is then the only restriction on the further arbitrary profiles for density $\rho(r)$, pressure $p(r)$, magnetic field components $B_0(r)$, $B_z(r)$, and sheared flows $v_0(r)$ and $v_z(r)$. Note that this ideal MHD equilibrium relation does not include the (small) radial accretion flow onto the central mass $M_*$ itself: we intend to perform a complete stability analysis about an exact stationary, stratified disk. In the equilibrium, the main force balance is one between centrifugal forces, gravity, pressure gradients and the Lorentz force. In this so-called cylindrical disk limit, one can analyse all the normal modes using the equation of motion for the Lagrangian displacement field $\xi(r, \theta, z, t)$, as derived by Frieman & Rothenberg (1960). Exploiting the 1D nature of the equilibrium, eigenoscillations are sought for modes with prescribed Fourier modes about and along the symmetry axis as in $(\xi_\ell(r), \xi_\ell(r), \xi_\ell(r), \exp i(\ell \theta + kz - \omega t))$. It is then possible to manipulate the entire set of linearized MHD equations to a single second order ODE. This ODE has singularities, which are known to constitute continuous ranges $\omega(r)$ of (purely real) eigenfrequencies. These correspond to stable, oscillatory perturbations on single flux surfaces (fixed radii in this 1D configuration), and the Alfvén continuum modes are analytically given by $\omega^2 - F^2 = 0$, where $F = mB_0/r + kB_\theta$ and where we introduced the Doppler shifted frequency $\tilde{\omega} = \omega - m v_\theta / r - kv_z$. The slow continuum modes are computed from $\rho \omega^2 (\gamma p + B^2) - \gamma p F^2 = 0$, where $\gamma$ is the ratio of specific heats. Due to the radial variation of the equilibrium and the Doppler shift, four continuous ranges of real eigenfrequencies are found. These, together with the (fast magnetosonic) cluster frequencies at $\tilde{\omega} = \pm \infty$ form the basic organizing structure for the MHD spectrum, consisting of both the continuous frequency ranges, as well as (possible cluster sequences of) discrete modes. In a series of works exploiting both analytical and numerical means to study the full MHD spectrum of radially stratified disks in the cylindrical limit, we have thus far:

- presented the governing equations for all linear waves and instabilities of accretion disks, identified the MRI as a cluster sequence of discrete modes associated with the slow continuum in weakly magnetized disks, and gave clear evidence of a richer variety of instabilities accessible to the plasma than previously realized (Keppens, Casse, & Goedbloed, 2002).
- revisited the axisymmetric ($m = 0$) MRI in disks having both axial and toroidal magnetic field components. In the presence of a toroidal $B_\theta$ component, its character is that of an overstable wave mode, which persists to exist for field equipartition field strengths (Blokland et al., 2005).
- quantified the growth rate of $m = 0$ convective MRI instabilities, in particular for thick disk models where convective instabilities can be enhanced in growth rate by magneto-rotational effects. This exploited a local dispersion equation which can be derived from the general formalism mentioned above by WKB means (van der Swaluw et al., 2005). It remains of interest to analyse in particular the many possibilities for non-axisymmetric ($m \neq 0$) modes in radially stratified disks, further adopting this ‘cylindrical disk’ approach.

Accretion tori: equilibrium. The analysis of the Frieman & Rothenberg (1960) equation governing normal modes becomes much more involved when axisymmetric, 2.5D equilibria for accretion tori are spectrally diagnosed. In fact, even when the equilibrium rotation is purely toroidal, the computation of the exact ideal MHD equilibrium itself is non-trivial. From
axisymmetry and the solenoidal constraint, one can write the equilibrium field as

\[ \mathbf{B} = \frac{1}{R} \mathbf{e}_\varphi \times \nabla \psi + B_\varphi \mathbf{e}_\varphi, \tag{2} \]

introducing the poloidal flux \(2\pi \psi\). We here use \((R, Z, \varphi)\) to denote cylindrical coordinates. Force balance to achieve equilibrium leads to the extended Grad-Shafranov equation

\[ R^2 \nabla \cdot \left( \frac{1}{R^2} \nabla \psi \right) = -I \frac{dI}{d\psi} - R^2 \frac{\partial p}{\partial \psi}. \tag{3} \]

In this PDE, we have a flux function \(I(\psi) = RB_\varphi\) and the pressure \(p = p(\psi; R, Z)\). Along the poloidal field lines (contour lines of \(\psi(R, Z)\)), additional relations hold involving the gravitational stratification. Once such equilibrium is found by simultaneously satisfying these constraints, a spectral analysis can again be performed. Figure 1 shows a particular example of an exact MHD equilibrium for an axisymmetric accretion torus with only toroidal rotation. A contour plot of the pressure distribution through a poloidal, assumed circular, cross-section of the torus is shown. The central gravitational mass about the plasma which this torus is rotating is to the left of the figure, while the central gravitational mass of the torus is shown. The central gravitational mass about the equilibrium field as

\[ \mathbf{B} = \frac{1}{R} \mathbf{e}_\varphi \times \nabla \psi + B_\varphi \mathbf{e}_\varphi, \tag{2} \]

Due to their singularly localized nature. Their persistence at small density stratification provides an excellent possibility for the development of turbulence in already saturated, initially weakly magnetized, disk evolutions.

**References.**
Friedman, E., Rotenberg, M. 1960, Rev.Mod.Phys. 32, 898
Velikhov, E.P. 1959, Sov. Phys. JETP, 36(9), 995

**Accretion tori: MHD spectroscopy.** Once an exact ideal MHD, axisymmetric equilibrium is found, one can start to compute all normal modes supported by the system. We progressed with both analytical and numerical approaches (with the PHOENIX code, Blokland et al., 2007) to demonstrate the existence of several new (i.e. completely unrelated to the conventional MRI mode) instabilities in multi-dimensional, magnetized disks. So far, we concentrated mainly on the distinct possibility that the flux-surface localized modes represented by the Alfvén and slow continua become unstable. As mentioned above, for a disk in the cylindrical limit, these continua always represent stable modes. Only by incorporating mode coupling due to the 2D nature of the equilibrium itself, can continuum modes turn overstable or unstable. Summarizing two recent findings, we showed that

**•** accretion tori with purely toroidal equilibrium flows can be unstable due to ‘convective continuum instabilities’ (CCI). When the density stratification corresponds directly to that given by poloidal flux contours, i.e. when \(\rho(\psi)\), these instabilities may arise. Figure 1 shows, for the equilibrium torus shown at left, the distribution of the continuum eigenmodes throughout the complex eigenfrequency plane. The overstable modes found are convective continuum modes with growth rates that are a significant fraction of the Alfvén frequency (Blokland, Keppens, Goedbloed, 2007).

**•** accretion tori with both toroidal and poloidal flows are prone to instability whenever the poloidal flow exceeds the slow magnetosonic speed, i.e. for equilibria in the slow elliptic flow regime. These trans-slow Alfvén continuum modes (TSAC) derive their existence from a six-mode coupling between forward and backward Doppler shifted Alfvén branches at fixed poloidal mode, and its two sidebands of forward and backward Doppler shifted slow branches (Goedbloed et al., 2004).

Future work will need to investigate the (many) possibilities for discrete modes that are driven unstable due to interacting gravitational, rotational, and thermodynamic forces. The unstable continuum modes discovered thus far provide an excellent possibility for the development of turbulence in stratified disks, due to their singularly localized nature. Their persistence at equipartition field strengths is crucial to invoke their possible role in sustaining MHD turbulence in already saturated, initially weakly magnetized, disk evolutions.

**References.**
Friedman, E., Rotenberg, M. 1960, Rev.Mod.Phys. 32, 898
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